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DEPARTMENT OF ECONOMICS

Is This Obvious? Screening for Important Contributions in Science.

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FACULTY OF ECONOMICS AND BUSINESS



DISCUSSION PAPER SERIES DPS 24.12

October 2024

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September 2024

Abstract

This paper studies how to reward obvious and non-obvious ideas differently when researchers have private information about obviousness, and non-obvious ideas are important but particularly costly to develop. Because obvious ideas might still value rewards like publications or patents, screening with fees or delay is not possible. Instead the planner must use costly scrutiny. We show that when scrutiny is imperfect, there is a tension between screening for obviousness and the efficient development of ideas: non-obvious ideas are less developed, and obvious ideas sometimes more developed, than they would be in the full-information optimal allocation. The constrained efficient allocation has features that look like the publication process, including tiers of journals and a top tier that is very exclusive. Changes in scrutiny costs predict changes to journal articles that are in line with empirical evidence on top journals.

Keywords: economics of science, academic publishing, screening, obviousness

JEL classification: D82, D86, O31

*We are grateful to Amirreza Ahmadzadeh, Carles Mañó Cabello, Natalia Fabra, Joshua Gans, Emeric Henry, Angel Hernando Veciana, Yassine Lefouili, Jorge Lemus, Igor Letina, Geert van Moer, Mariann Ollar, Jens Prüfer, Johannes Schneider, Vasiliki Skreta, Balazs Szentes, John Turner, and Juuso Välimäki for helpful comments. We also benefited from conference and seminar participants at Helsinki, Tilburg, Boston, Bath, Leuven, Urbana-Champaign, Groningen, Barcelona, Rome, Madrid, and the 2020 Economics of Innovation Workshop (online). All errors are our own.

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1 Introduction

In science, researchers are rewarded for developing non-obvious ideas. Examples include publishing academic papers, funding grant proposals, and issuing patents. It is thought that the development of truly non-obvious ideas is key to innovation and knowledge accumulation. At the same time, the goal is to reward non-obvious ideas that are also well developed into clear, useful knowledge. This paper studies the optimal rules for development of non-obvious ideas, and trade-offs that arise in screening for obviousness and inducing development. We show that screening and development may be at odds: it is often worth sacrificing thorough development in order to screen at lower cost. In other words, an excessively high standard of development may hinder the screening process for obviousness.

As in much of the literature on scientific publication and patenting, we assume that the researcher has private information about whether the idea is obvious. As such, designing a system to incentivize the researcher to reveal this information can economize on costly scrutiny of the submission. In contrast to much of the literature, we assume that without scrutiny, an obvious idea values acceptance as much as a non-obvious one. This implies that the researcher's information is not readily extracted by making submission have costs, either monetary or otherwise. Instead, screening relies on scrutiny.

We study constrained efficient allocations via a planner's problem and compare them to observed practice in academic publishing. The optimal structure typically involves a two-tiered system: a top tier which faces more scrutiny, but only receives submissions from researchers who think their idea is sufficiently non-obvious, and a lower tier that faces lower scrutiny. The higher tier has a very high bar, in the sense that submissions are rejected except for the most informative of signals. The top tier has a natural interpretation in the publication example as a highly prestigious journal.

We emphasize the connection, and sometimes the tension, between scrutiny for obviousness and a complete development of the idea. To better understand this tension, consider a researcher that wants to test a general mechanism (the idea) with specific tests (the development). Each test that confirms the result in a specific context further develops the case for their mechanism; we think of this as choosing a greater amount of development. To the extent that the standard for the number of different tests rises, and to the extent that it is easier to run tests on obvious ideas (or cheaper in expectation, because they are more likely to be confirmed once run), setting a high standard for thorough development is relatively attractive to obvious ideas, which can achieve the required standard at lower cost. A high standard of development therefore makes it more costly to screen non-obvious ideas.

We show that development effort for non-obvious ideas is distorted downward below the first-best level. In order not to over-reward obvious ideas, the planner must sacrifice development of non-obvious ideas. Moreover, in some cases the planner must bias the development effort for obvious ideas up from the first best so as to ensure incentive compatibility for non-obvious ideas. This may explain why we observe effort on obvious ideas that does not seem justified by their social benefit, relative to more important contributions.

The comparative statics of the model generate a number of predictions that are consistent with observed trends in academic publishing. There is reason to believe that the cost of scrutinizing obviousness has gone down in recent decades, as improvements in information and communication technologies, including electronic access to bibliographic databases and full-text journal articles as well as more powerful search tools, have made it easier for reviewers to compare submitted papers to the state of the art. The model predicts that a decrease in scrutiny costs would lead to enhanced scrutiny and higher standards for thorough development at top journals. This is in line with evidence that published articles have become longer (Card and

DellaVigna, 2013); casual observation also suggests that they contain more extensions or robustness checks than a few decades ago.

If, as seems plausible, more scrutiny leads to lower acceptance rates, then the combination of lower acceptance rates and higher development standards implies that authors of non-obvious ideas must be compensated through higher rewards. This conforms with the increased importance of publishing in the top-five economics journals for tenure and promotion decisions (Heckman and Moktan, 2020).

The model also has predictions about the way different authors are treated. A submission from an author with a higher probability of generating non-obvious submissions is asked for more development. This is consistent with evidence in Card and DellaVigna (2020) that more prolific authors are held to higher standards in the publication process.

Another application is to patents. Our model speaks to two main concerns that have recently been the focus of the debate on "bad patents": obvious and vague patents. Vague patents often violate the utility requirement whereby a patent must show specific use, and not a general concept. We think of this as incomplete development.[1] Obvious patents violate the non-obviousness requirement, whereby the invention cannot be obvious to a person of ordinary skill in the art. The granting of obvious patents has often been highlighted as a problem with the patent system (Lemley et al., 2005; Mandel, 2008). While the two concerns – vagueness and obviousness – are usually discussed in isolation, we show that they are linked. Our analysis suggests that there may be an important trade-off between them.

[1]In many patent statutes, including the US, a utility requirement specifies that an idea be developed to the point of use in order to get a patent, for instance specifically in the US statute an application must be "useful and not abstract." The Australia government Patent Manual of Practice and Procedure defines vague patents as ones for which "the description gives merely an indication of the full breadth or scope of the invention, but no, or inadequate, directions of how to put it into practice" (https://manuals.ipaustralia.gov.au/patent/2.11.7.9a-broad-or-speculative-claims).

Although the US patent system has a single tier, many other patent systems around the world have two tiers, with a regular patent coexisting alongside a so-called utility model; examples of the latter include the German "Gebrauchsmuster" and the Australian "innovation patent." The top-tier patent offers longer protection (twenty years) and is scrutinized for patentability by examiners at the patent office, whereas the lower-tier utility model offers shorter protection (typically around ten years) and is issued without substantive examination.[2] This pattern is in line with our predictions. From a normative point of view, our analysis suggests that the efficiency of the U.S. patent system could be enhanced by introducing a second tier.

The paper also provides theoretical insights that are relevant outside the context of the specific applications we focus on. In our model, unlike in most of the literature, the principal may optimally distort both the low type's and high type's output. Moreover, both distortions reduce the wedge between the two types' outputs. This contrasts with standard adverse-selection models where typically only one type's output is distorted, and usually in such a way that the wedge between high and low-type outputs increases. Another noteworthy feature of our analysis is that the set of binding constraints tends to change with the level of scrutiny. In particular, for the high type, either only individual rationality, or only incentive compatibility, or both may be binding at the optimum.

Related literature. In modelling the academic publishing process, many models follow Leslie (2005), who shows that when submitters have information about their quality, fees can help to extract that information. Azar (2007) and Cotton (2013) extend this idea to fees and delay. Cotton (2013) shows that in a heterogeneous population of submitters, it is often optimal to use both instruments. By contrast, we explicitly model the scrutiny process

[2]See https://www.wipo.int/patents/en/topics/utility_models.html.

and consider the possibility of linking rewards from publication to the level of scrutiny, in order to address new questions related to journal quality and scrutiny strength. Fees are not a useful sorting device in our setup. Perhaps the closest paper to ours in this space is Muller-Itten (2022), who studies competition between journals (gatekeepers) with asymmetric information as we have and shows that excessive delay can improve efficiency by sorting, beyond scrutiny. We model optimal design of scrutiny as a contracting problem and draw out the tension between obviousness and development.

Our paper is more generally related to the literature on science processes. Our focus on incentives for developing non-obvious ideas is similar to Carnehl and Schneider (2023) and Veugelers et al. (2022), who study incentives for novelty in science. Unlike us, however, they do not consider a contracting problem. Bobtcheff et al. (2017) share our focus on trade-offs associated with the development of ideas. While we highlight the trade-off between development and screening, they emphasize how developing ideas more thoroughly may raise the probability of being preempted by another researcher.

Our focus on scrutiny and thus scientific peer review shares some features with models of the refereeing process (Ellison, 2002; Hirshleifer, 2015; Baghestanian and Popov, 2018). Baghestanian and Popov (2018) have a model with both effort and ability, but additively separable in one dimension. Therefore they do not have the tension that we highlight.

Another recent strand of the literature has focused on incentives to distort research findings. These include Henry and Ottaviani (2019), Libgober (2022), Andrews and Shapiro (2021), Frankel and Kasy (2022), Abadie et al. (2020), Yoder (2022) and Furukawa (2019), which are environments where the value of a potential contribution is known, and a researcher can manipulate the content of the evidence to influence beliefs about the state of the world. We abstract from this element and model scrutiny in a reduced-form

way, in order to highlight other issues in the design of incentives.[3]

Apart from the literature on science, our paper is also related to the patent literature. Much of that literature, dating to Mill (1848), assumes that private and social value of an idea are aligned, so that scrutiny is unnecessary; screening can occur with fees. This idea spawned the mechanism design approach to patents followed by Scotchmer (1999), Cornelli and Schankerman (1999), Hopenhayn and Mitchell (2001), and Hopenhayn et al. (2006), among others. These papers exploit the standard monotonicity assumption in screening.While we also follow the mechanism-design approach, we dispense with the monotonicity assumption to focus on the role of scrutiny for screening.

While the literature on patent screening has recognized the importance of scrutiny, many papers have adopted a game-theoretic approach. Perhaps the two closest papers are Schankerman and Schuett (2022) and Atal and Bar (2014). Schankerman and Schuett (2022) study the current system theoretically as well as quantitatively and simulate counterfactuals. They have a similar assumption that patents may be valuable privately without having social value. Atal and Bar (2014) study a two-tiered patent system with exogenous scrutiny and patent fees.[4] Neither paper studies optimal design, nor do they incorporate vagueness, another important element of problematic patents.[5]

[3]A related issue is the sharing of credit in academic teams, especially through name order (Engers et al., 1999; Ray and Robson, 2018; Ozerturk and Yildirim, 2021).

[4]Other papers such as Langinier and Marcoul (2016), Caillaud and Duchene (2011), Schuett (2013a) and Schuett (2013b) have studied examination, but in the context of examiner incentives. Llobet et al. (2021) study scrutiny in the presence of judges and patent trials, focusing on the incentives of the judges. Farrell and Shapiro (2008) consider the question of probabilistic ("weak") patents and their impact on the patent system.

[5]At a technical level, the paper relates to the literature on costly state verification, which started from the work of Townsend (1979). Recent contributions include Ben-Porath et al. (2014), Mylovanov and Zapechelnyuk (2017), Halac and Yared (2020), Li (2020), and Ahmadzadeh and Waizmann (2024). In this literature, the principal can learn the agent's type perfectly by paying a fixed cost. By contrast, we consider an imperfectly

The remainder of the paper is organized as follows. Section 2 introduces the model and derives the complete-information benchmark. Section 3 contains the core of the paper, starting with an analysis of the general structure of constrained optimal allocations in Subsection 3.1 before turning to the distortions in development effort they involve in Subsection 3.2 and to their comparative statics properties in Subsection 3.3. Section 4 discusses some key assumptions while Section 5 concludes. All proofs are relegated to the Appendix.

2 Model

A researcher with an idea of type $\theta \in \{l, h\}$, where $l < h$, is considering both how much effort e to expend developing the idea and where to submit the finished work. The type of the idea θ corresponds to its degree of non-obviousness and is distributed according to $\alpha(\theta)$. With a slight abuse of notation, we sometimes use $\alpha(h) = \alpha$ and $\alpha(l) = 1 - \alpha$. We refer to ideas of type l as *obvious* and to ideas of type h as *non-obvious*.

From society's point of view, all ideas are valuable, but the marginal social value of effort is higher for non-obvious ideas than for obvious ones. We assume that the social value of effort e expended on an idea of type θ is θe, and provide further interpretation of this assumption below. The key feature this captures is that society relies more heavily on the researcher's development effort when the idea is non-obvious, as it will be harder for others to unlock the social benefits associated with the idea.[6]

Although all that is needed for the formal results is that the social value

informative signal whose precision is a choice variable for the principal. The only other papers we are aware of that consider imperfectly informative signals are Cabrera (2018) and Ball and Kattwinkel (2022).

[6]Although the relationship between θ and effort in generating social returns is proportional, we allow the cost of providing that effort for different θ to be more flexible; see below.

is θe, for interpretation we use the following explicit structure. Suppose all ideas have the potential to generate social benefits normalized to one. Let $\omega \in \{0, 1\}$ be a state of the world that is unobservable to everyone. If $\omega = 1$, unlocking the social benefits requires effort, while if $\omega = 0$, it does not; this is meant to match the notion that some insights can be uncovered for free. Assume $\theta = \Pr(\omega = 1|\theta)$: that is, θ is the probability that generating social benefits requires effort. Assume also that the researcher's effort $e \in [0, 1]$ is the probability that the researcher unlocks the social benefits when $\omega = 1$. When $\omega = 0$, the idea always generates social benefits regardless of the researcher's effort. The expected social value of effort for an idea of type θ is $\theta e + 1 - \theta$. Because $1 - \theta$ is constant, it does not matter for the analysis and will be suppressed in what follows.

Whether society will eventually receive the benefits of the effort is unknown at the time the research is submitted for publication. This reflects the fact that the social benefits from scientific advances materialize with delay.

Effort costs $c(\theta, e)$ are increasing and convex in e, with $c(\theta, 0) = 0$, and satisfy the Inada condition $c_e(\theta, 0) = 0$. Non-obvious ideas are more costly to develop, i.e., $c_\theta > 0$. This assumption embodies the notion that, if an idea is less obvious, then (at least in expectation) more experimentation is required to bring it to fruition. We also impose the following assumption:

Assumption 1. *The cost function is multiplicatively separable:* $c(\theta, e) = \kappa(\theta)\psi(e)$.

Although some of our results would go through without multiplicative separability, we maintain this assumption throughout the analysis in order to streamline the exposition.[7] Notice that, together with $c_\theta > 0$, Assumption 1 implies $c_{e\theta} > 0$.

[7] Our previous assumptions imply $\kappa' > 0$ and $\psi', \psi'' > 0$.

We assume that e is observable to the planner whereas θ is not. The submission can be judged in terms of whether the analysis is complete and correctly done; the importance of the contribution, in terms of generating non-obvious insights, is what is uncertain. The planner has two ways to learn about θ. The first is to ask the researcher, but this opinion is naturally conflicted by incentives. Additionally, the planner has a technology to scrutinize ideas. This could include asking an opinion of an outsider like a referee. By spending $x \geq 0$ on scrutiny, the planner can generate a signal drawn from a set of n possible realizations according to the distribution $p_i(x, \theta)$, where $i \in \{1, \ldots, n\}$ indexes the signal realization. This corresponds to the refereeing or editorial process; the level of scrutiny depends, for example, on how many referees are enrolled to review a paper or how much time and effort the editor and referees spend on the review. We assume that p_i is twice continuously differentiable and that, in the absence of scrutiny ($x = 0$), the signal is uninformative: $p_i(0, \theta) = p_i(0, \hat{\theta})$ for all $\theta, \hat{\theta}$ and all i.

The critical feature of signals in generating incentives is their likelihood ratio, given by $p_i(x, l)/p_i(x, h)$, since it determines the inference that can be drawn from that signal about θ. Assume without loss of generality that, for $x > 0$, no two signals have the same likelihood ratio.[8] Let k be the signal with the lowest likelihood ratio, i.e.,

$$k = \arg \min_{i \in \{1,\ldots,n\}} \frac{p_i(x, l)}{p_i(x, h)}.$$

We impose the following assumption:

Assumption 2. *The likelihood ratio $p_k(x, l)/p_k(x, h)$ is decreasing in x, with $\lim_{x \to 0} \partial \left(p_k(x, l)/p_k(x, h) \right) / \partial x = -\infty$.*

Assumption 2 says that scrutiny makes signal k more precise, in the sense of decreasing the likelihood ratio. In other words, scrutiny is assumed to

[8] When two signals have the same likelihood ratio, we can re-define them as being the same signal and all of the formal results are unchanged.

improve the relative chances of obtaining signal k for high types compared to low types. The assumption also introduces an Inada condition on the sensitivity of the likelihood ratio to changes in scrutiny at $x = 0$.

Scrutiny can make it either easier or harder for high-type papers to obtain signal k (i.e., $p_k(x, h)$ can be increasing or decreasing in x); Assumption 2 is compatible with both. For a concrete example of how a more precise signal could nevertheless be harder to obtain even for high types, suppose applying more scrutiny requires enlisting more referees, and that the lowest-likelihood-ratio signal is that all referees recommend acceptance.[9] If each referee independently recommends acceptance with probability q_θ, where $q_h > q_l$, then when x referees provide opinions the likelihood ratio is $(q_l/q_h)^x$, while the probability of obtaining signal k for high types is q_h^x. Both decrease with x.

The planner motivates researchers through publication in journals. The reward a researcher receives from publication should be interpreted broadly; it may encompass both monetary aspects (improved lifetime earnings) and non-monetary aspects the researcher values (e.g., prestige). We assume that the planner controls the reward $S \geq 0$ associated with publication in a given journal.[10]

Because the reward S is related to private benefits from publishing, which are subject to some underlying scarcity, allocation of journal space is socially costly. We assume this cost is proportional to the reward it generates. Therefore, giving S units of reward costs the planner δS to provide. The planner's reward when giving reward S to a researcher of type θ who expends effort e

[9] Akerlof (2020) argues that, because acceptance in journals is decided by a committee of editors and reviewers, the scarcity of rewards implies that acceptance typically requires all committee members to approve.

[10] Alternatively, the planner may take the rewards associated with publication in a set of available journals as given. If publication in journal j generates a reward S^j for the researcher, then the planner chooses a mixture $\{\mu^j\}$ over journals such that $S = \sum_j \mu^j S^j$. Our assumption that the planner controls the reward then requires that the set of journals and the range of associated rewards are rich enough.

is therefore $\theta e - \delta S - c(\theta, e)$, while the researcher's payoff is $S - c(\theta, e)$.[11]

As a benchmark, suppose that the planner could observe θ, i.e., that the scrutiny technology were infinitely accurate at no cost. Participation by the researcher requires $S \geq c(\theta, e)$, and since rewards are costly this constraint binds at the optimum. Then the planner solves, for each θ,

$$\max_e \theta e - (1 + \delta)c(\theta, e). \tag{1}$$

Let $e^*(\theta)$ denote the solution to this problem, implicitly defined by the first-order condition, $\theta = (1+\delta)c_e(\theta, e^*(\theta))$, which under our assumptions is both necessary and sufficient.

We assume that non-obvious ideas are more *important* in the sense that, under complete information, more effort should be expended on ideas with higher θ. This requires that $de^*(\theta)/d\theta > 0$. The following assumption formalizes this requirement in terms of the cost function c.

Assumption 3. $\partial(c_e(\theta, e)/\theta)/\partial\theta < 0$.

Assumption 3 can be rewritten as $c_{e\theta} < c_e/\theta$. Intuitively, it says that the marginal cost of effort, c_e, cannot increase too fast with θ relative to the marginal benefit of effort, given by θ.

We study the planner's optimal allocation under asymmetric information about obviousness. We mostly think of this as a positive prediction: institutions tend toward these constrained-efficient allocations in the long run. Alternatively, but less of our focus, one can think of these as normative recommendations for an efficient system.

[11] Our model abstracts from the dissemination benefit that the planner might get by publicizing novel science. We view the rise of dissemination sources like arXiv as making the publication process more about rewarding science and less about dissemination. Dissemination benefits could be incorporated into the model by making the cost of publication depend on type, and be lower for non-obvious ideas.

3 Analysis

In order to construct constrained efficient allocations, we invoke the Revelation Principle and restrict attention to a direct revelation mechanism which asks the researcher to report their type θ and specifies a required effort $e(\theta)$, a level of scrutiny $x(\theta)$, and a reward structure $S(\theta) = (S_1(\theta), \ldots, S_n(\theta))$, where $S_i(\theta)$ is the reward associated with the journal the researcher's work is published in if they report θ and signal i is realized. When the researcher reports truthfully, their expected payoff is $\sum_i p_i(x, \theta) S_i(\theta) - c(\theta, e)$.

The planner solves

$$\max_{e(\theta), x(\theta), S(\theta)} \sum_{\theta} \left(\theta e(\theta) - \delta \sum_{i=1}^{n} p_i(x(\theta), \theta) S_i(\theta) - c(\theta, e(\theta)) - x(\theta) \right) \alpha(\theta) \quad (2)$$

subject to the incentive compatibility (IC) and individual rationality (IR) constraints:[12]

$$\theta = \arg\max_{\hat{\theta}} \sum_{i=1}^{n} p_i(x(\hat{\theta}), \theta) S_i(\hat{\theta}) - c(\theta, e(\hat{\theta})) \quad \forall \theta$$

$$\sum_{i=1}^{n} p_i(x(\theta), \theta) S_i(\theta) - c(\theta, e(\theta)) \geq 0 \quad \forall \theta.$$

A key feature of this problem is that assigning different development efforts to different types of researchers has only a limited ability to screen without scrutiny; if there is no scrutiny for either low or high types ($x(l) = x(h) = 0$), the first-best efforts are not implementable. Adding up the IC constraints of types l and h, we obtain the following necessary condition for incentive compatibility when $x(\theta) = 0$ for all θ:

$$c(l, e(h)) - c(l, e(l)) \geq c(h, e(h)) - c(h, e(l)). \quad (3)$$

[12] Note that, unlike in other models of mechanism design with costly verification (see, e.g., Ahmadzadeh and Waizmann, 2024; Ball and Kattwinkel, 2022), there is no need for ex-post participation constraints in our setting. This is because of our assumptions that rewards must be non-negative ($S_i(\theta) \geq 0$) and that development effort (e) is chosen before the planner's signal about the researcher's type is realized, both of which are plausible in the science application that is our focus.

Because, by assumption, $c_{e\theta} > 0$, (3) holds if and only if $e(l) \geq e(h)$. But under Assumption 3, the first-best efforts are such that $e^*(h) > e^*(l)$, and thus fail to satisfy (3). The intuition is that, when $x = 0$, the signal distribution does not depend on the researcher's type, so their expected reward depends only on their reported type and not on their true type:

$$\sum_{i=1}^{n} p_i(0, \theta) S_i(\hat{\theta}) = \sum_{i=1}^{n} p_i(0, \hat{\theta}) S_i(\hat{\theta}) \text{ for all } \theta, \hat{\theta}.$$

Thus, incentive compatibility must rely on cost differences, and since the low type's marginal cost of effort is smaller than the high type's, only allocations in which low-type effort exceeds high-type effort can be implemented.[13]

3.1 Tier structure

We now consider the possibility of positive scrutiny for high types, $x(h) \geq 0$, and show that it gives rise to a structure that resembles a journal system. For ease of exposition, we restrict attention to allocations in which $x(l) = 0$, i.e., researchers reporting to be low types receive no scrutiny. One justification is that high types may be able to impersonate low types at no cost, perhaps because they can submit research that studies a special case of their idea, thus making it appear more obvious than it is.[14] Section 4 provides further discussion of situations where $x(l) = 0$ is optimal.

The following result characterizes second-best allocations. They resemble a two-tiered journal system, with a top tier of journals that have a higher standard of acceptance.

Proposition 1. *The constrained-optimal allocation has $x(h) > 0$, $e(h) > 0$, and $e(l) \geq e^*(l)$. It involves a reward structure for high types that is highly*

[13] This is an instance of what the contract-theory literature, following Caillaud et al. (1988), has referred to as non-responsiveness.

[14] Formally, high types can report $\hat{\theta} = l$ and obtain the same signal distribution as low types, $p_i(x(l), l)$. They retain the cost function $c(h, e)$, however.

skewed: $S_k(h) > 0$ *and* $S_i(h) = 0$ *for* $i \neq k$.

Proposition 1 says that the optimal contract always involves strictly positive scrutiny for high types and strictly positive development efforts for both types. In addition, all rewards to the high type are loaded onto the signal with the lowest likelihood ratio (k). The proof shows that the low type's IC constraint is binding at the optimum: if it was not, the planner could either lower the reward given to low types or reduce scrutiny of high types, thus saving on costs. This implies that the expected reward the planner must give to low types is determined by their payoff from misreporting, i.e., claiming to be the high type.

Conditional on giving an expected reward of fixed size to the high type, the low type's payoff from misreporting is minimized by structuring the high type's reward so as to be highly skewed, with only signal k being rewarded. The proof uses the Kuhn-Tucker conditions of the minimization problem to show that the non-negativity constraint $S_i(h) \geq 0$ can be slack for at most one i.[15] The intuition is that the likelihood ratio reflects how informative signal i is about type. The signal with the lowest likelihood ratio is the one that is least likely to be realized for type l relative to type h. Hence, loading the high type's reward entirely on that signal is the best way to make misreporting unattractive for the low type.

How low types' reward is allocated across signals is not pinned down: since low types face no scrutiny, any reward structure that gives the same expected reward is equivalent. Thus, it is without loss to give them a reward that is independent of the realized signal: $S_i(l) = S_l$ for all i. With a small amount of risk aversion on the side of low-type researchers, it would be strictly optimal to structure rewards in this way.

To sum up, the optimal contract has features that resemble a journal

[15]The fact that $e(h) > 0$ is optimal implies that $S_i(h)$ must be strictly positive for at least one i.

system, with a top tier of journals that publish better ideas (in the sense of being less obvious) and are more difficult to get into. Submissions to top-tier journals are accepted only when the best signal is realized, whereas submissions to lower-tier journals are always accepted. Below we show that if scrutiny is not too costly, the optimal allocation also involves high types developing their ideas more fully, which implies higher rewards at the top journals.

3.2 Effort Levels and Effort Distortions

In this section we characterize the effort distortions that are generated by asymmetric information and show how they relate to observed outcomes in the publication process.

To characterize the solution to the planner's problem, we first derive the optimal allocation for a given level of high-type scrutiny, $x(h) \equiv x \geq 0$. We then perform comparative statics on x. Since scrutiny is endogenous, one should think of changes in x as being induced by changes in the (exogenous) cost of scrutiny. We make this argument more formally below.

The analysis in the previous section allows us to simplify the planner's problem. Proposition 1 implies that it is optimal to structure high-type rewards such that only the lowest-likelihood-ratio signal, k, is rewarded. Let the probability of that signal, given x, be $p_h \equiv p_k(x, h)$ for the high type and $p_l \equiv p_k(x, l)$ for the low type. Similarly, denote the reward following a report of h when the signal is k by $S_h \equiv S_k(h)$. To harmonize notation, we will also use $e(h) = e_h$ and $e(l) = e_l$.

It is convenient to write the planner's problem in terms of the rents that accrue to each type rather than in terms of the rewards. Letting $V_l \equiv S_l - c(l, e_l)$ and $V_h \equiv p_h S_h - c(h, e_h)$ denote each type's rent, we can rewrite the

planner's problem (for given scrutiny) as

$$\max_{e_l,e_h,V_l,V_h} W = \alpha(he_h-(1+\delta)c(h,e_h)-\delta V_h)+(1-\alpha)(le_l-(1+\delta)c(l,e_l)-\delta V_l) \quad (4)$$

subject to

$$V_h \geq 0 \quad (5)$$

$$V_l \geq 0 \quad (6)$$

$$V_h \geq V_l - (c(h,e_l) - c(l,e_l)) \quad (7)$$

$$V_l \geq (1-\rho)(V_h + c(h,e_h)) - c(l,e_h), \quad (8)$$

where $\rho \equiv (p_h - p_l)/p_h$. Eqs. (5) and (6) are the IR constraints while Eqs. (7) and (8) are the IC constraints for high and low types, respectively.

Notice that the planner's problem for a given x is entirely determined by the value of ρ for that x and does not depend on p_l and p_h separately. Assumption 2 implies ρ is increasing in x, so we speak of ρ directly to describe scrutiny.

To begin, we show that if scrutiny is sufficiently intense, the first best can be implemented.

Lemma 1. *Suppose $\rho \geq \rho^*$, defined by*

$$(1-\rho^*)c(h,e_h^*) = c(l,e_h^*).$$

Then, the planner implements the first-best research efforts: $e_l = e_l^*$ and $e_h = e_h^*$.

When $\rho \geq \rho^*$, the top tier is sufficiently unlikely for obvious ideas to get into that the planner does not need to give up rents to keep them from submitting, even when non-obvious ideas get a reward large enough to compensate for full (efficient) development. Note that, by continuity, for ρ sufficiently

close to ρ^*, $e_h > e_l$, implying $S_h > S_l$: provided there is enough scrutiny, rewards are higher at the top-tier journal (conditional on acceptance).[16] Note also that the planner can achieve the first best without perfect scrutiny: Assumption 3 implies that $\rho^* < (h-l)/h$, which is the level at which planner and researcher would be equally informed.

For the remainder of the paper we focus our attention on $\rho < \rho^*$. The planner never scrutinizes beyond this point, as it does not change the planner's payoff except to increase scrutiny cost. For $\rho < \rho^*$, the first best is unattainable, and the optimal allocation will entail effort distortions. We start by considering the high type's effort. It is distorted downward: top-tier journals ask for less development than is efficient.

Proposition 2. *For $\rho < \rho^*$, the high type's effort is distorted below the first-best level: $e_h < e_h^*$.*

Proposition 2 shows that there is a tension between screening obvious ideas and obtaining efficient development of non-obvious ideas. The hard work required to develop non-obvious ideas to the first-best level calls for high rewards from top-journal publication. Without enough scrutiny, these high rewards also make it attractive for researchers with obvious ideas, which need to work less hard to attain any given level of development, to submit their papers to the top journal. To dissuade them from doing this, they must be compensated accordingly at the lower-tier journal, forcing the planner to give up costly rents. Because of the low types' cost advantage, decreasing the level of development required for non-obvious ideas and adjusting the high-type researcher's reward by the associated cost difference allows the planner to save on those rents.

To see why this is optimal, consider starting from an allocation with $e_h \geq e_h^*$, and marginally lowering e_h while keeping e_l and V_h fixed. This

[16] Suppose $e_h \geq e_l$. By the high type's IC, $p_h S_h \geq S_l + c(h, e_h) - c(h, e_l) \geq S_l$ and hence $S_h \geq S_l$, with strict inequality for $e_h > e_l$ or $p_h < 1$.

produces at worst a second-order loss in welfare from high types: if $e_h = e_h^*$, the marginal welfare effect is zero, while if $e_h > e_h^*$, it is strictly positive. At the same time, reducing e_h relaxes the low type's IC constraint. Formally, when the cost function is multiplicatively separable (Assumption 1), $(1-\rho)c_e(h, e_h) - c_e(l, e_h)$ has the same sign as $(1-\rho)c(h, e_h^*) - c(l, e_h^*)$, which is strictly positive for $\rho < \rho^*$ by the definition of ρ^*. Thus, higher effort by the high type increases the low type's payoff from misreporting. The low type's IC constraint must be binding for $\rho < \rho^*$; hence, distorting high-type effort down allows the planner to reduce low-type rents, producing a first-order welfare gain.

To further characterize the optimal contract, suppose we solve a relaxed problem where we impose only the high type's IR and the low type's IC, ignoring the other constraints. The solution to this relaxed problem has $V_h = 0$, $e_l = e_l^*$, $V_l = (1-\rho)c(h, e_h) - c(l, e_h)$, and $e_h = \tilde{e}(\rho)$ implicitly defined by

$$\alpha[h - (1+\delta)c_e(h, \tilde{e}(\rho))] = (1-\alpha)\delta[(1-\rho)c_e(h, \tilde{e}(\rho)) - c_e(l, \tilde{e}(\rho))]. \quad (9)$$

The low type's IR is always satisfied at this allocation for $\rho \leq \rho^*$. The high type's IC is satisfied if

$$(1-\rho)c(h, \tilde{e}(\rho)) - c(l, \tilde{e}(\rho)) \leq c(h, e_l^*) - c(l, e_l^*). \quad (10)$$

If, for some $\rho < \rho^*$, (10) holds, then the solution to the relaxed problem is also the solution to the full problem given a level of scrutiny ρ. We term such a case the *regular solution*.

The regular solution necessarily applies for ρ sufficiently close to ρ^*, as the left-hand side of (10) is zero at ρ^* while the right-hand side is strictly positive. Figure 1 shows a parametric example in which condition (10) holds for all $\rho \geq 0$. On the horizontal axis, the figure plots scrutiny for high types, ρ, while, on the vertical axis, it plots the constrained optimal research efforts

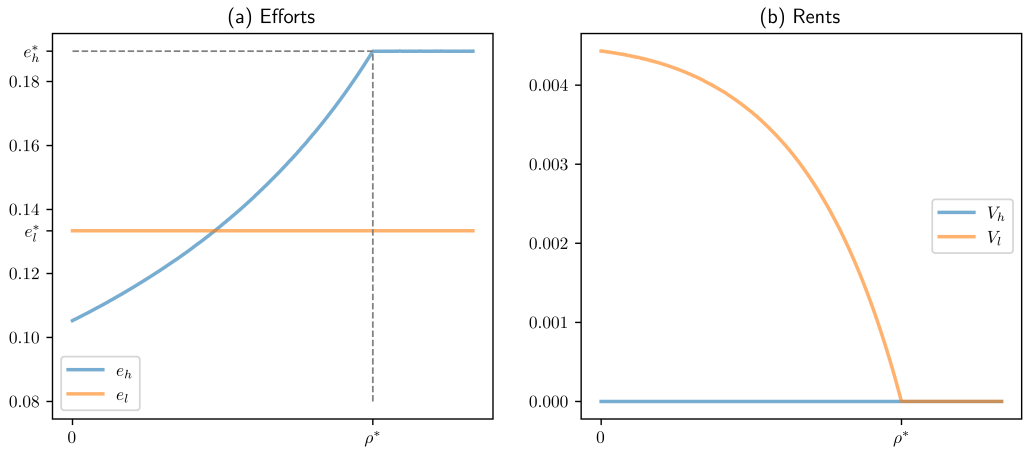

Figure 1: Research efforts and rents when the type-h IC is slack for all ρ

(panel (a)) and the resulting rents (panel (b)). The probability of the lowest-likelihood-ratio signal, k, is specified as $p_k(x,\theta) = q(x)\theta + (1-q(x))(1/n)$, with $q(x) = x/(1+x)$.[17] The cost function is specified as $c(\theta, e) = (\kappa+\theta)e^{\psi}$. The figure is based on the following parameter values: $\alpha = 0.05$, $\delta = 0.25$, $\kappa = 1$, $\psi = 2$, $n = 10$, $l = 0.5$, and $h = 0.9$.

As the figure shows, for $\rho \geq \rho^*$, the planner sets both research efforts at their first-best levels, and rents are zero. For $\rho < \rho^*$, the low type's effort remains first best, but the high type's effort is distorted down; as explained above, this reduces the low type's rent. The high type earns no rent. Perhaps surprisingly, e_h can be distorted downward so strongly that it ends up below e_l. This happens when there is little scrutiny (ρ close to zero). Intuitively, in

[17] This implies

$$\rho = \frac{q(x)(h-l)}{q(x)h + (1-q(x))/n},$$

which is monotonically increasing in x. Thus, it can be inverted to express x as a function of ρ:

$$x = \frac{\rho}{n[h(1-\rho) - l]}.$$

this example the share of high types is small (five percent). Thus, the cost of distorting their development effort is small, while the resulting savings in low-type rents are large.[18]

The case depicted in Figure 1 is not the only possibility; it can also be the case that (10) does not hold for all $\rho < \rho^*$. If the high type's IC is violated at the regular solution for some ρ, then an upward distortion of low-type effort is always optimal:

Proposition 3. *Suppose* (10) *does not hold at a given level of scrutiny* $\rho < \rho^*$. *Then the low type's effort is distorted above the first-best level:* $e_l > e_l^*$.

The case where the high type's IC is violated corresponds to a situation in which the researcher with a non-obvious idea would be tempted to submit to the low-quality journal. To make this unattractive, the low-quality journal must ask for extra effort, which is relatively easy for authors of obvious papers to provide, discouraging authors with non-obvious papers from meeting the submission standard of the low-quality journal.

To understand why the upward distortion of low-type effort is optimal, suppose the high type's IC is violated at the regular solution, i.e., for $e_l = e_l^*$ and $e_h = \tilde{e}(\rho)$. Consider an alternative allocation in which $e_h \neq \tilde{e}(\rho)$ while e_l remains at e_l^*. There are two cases. If the high type is earning a rent ($V_h > 0$), then distorting the low type's effort up causes only a second-order loss in welfare from low types while producing first-order gains in terms of lower rents. This argument is analogous to the one given for the downward distortion of high-type effort (Proposition 2).[19]

The more interesting case arises if the high type is not earning a rent ($V_h = 0$), i.e., if both IC and IR bind for the high type. In that case, it must

[18] One might have expected that the planner would rather shut down the high type, or have both types exert the same effort, than distort e_h below e_l. Neither of these turn out to be optimal, however. Shutting down the high type is not optimal because the marginal effect on the low type's rent from increasing high-type effort is zero at $e_h = 0$.

[19] The difference is that here, the distortion generates a reduction in *both* types' rents.

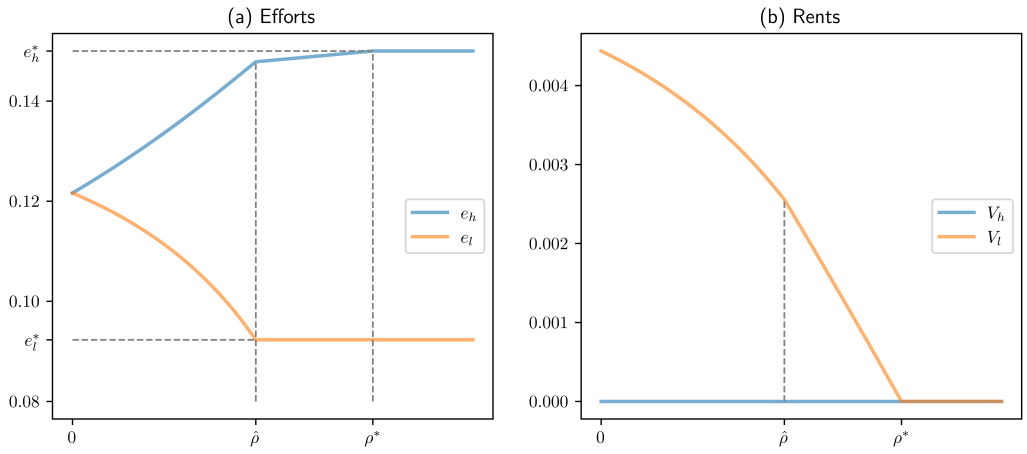

Figure 2: Research efforts and rents when the type-h IC binds for $\rho \leq \hat{\rho}$

be that $e_h < \tilde{e}(\rho)$. Marginally raising e_l above e_l^* then causes a second-order loss in welfare from low types but relaxes the high type's IC, allowing the planner to marginally raise e_h. Because the marginal effect of e_h on welfare net of low-type rents is zero at $\tilde{e}(\rho)$ (by definition), and $e_h < \tilde{e}(\rho)$, this produces a first-order gain.

Figure 2 depicts an example in which there exists some level of scrutiny $\hat{\rho} < \rho^*$ such that (10) is violated for all $\rho \leq \hat{\rho}$. The example is based on the same parameters as the previous one, except that $\alpha = 0.5$, $l = 0.3$, and $h = 0.6$. For high levels of scrutiny ($\rho \geq \rho^*$), both the high and the low type's efforts are efficient, and neither type earns a rent. For somewhat lower scrutiny ($\hat{\rho} \leq \rho < \rho^*$), the regular solution applies (IR binds for high types and IC for low types). High-type effort is biased down but low-type effort is efficient. For even lower scrutiny ($\rho < \hat{\rho}$), both the IC and IR constraint bind for high types. High-type effort is still biased downward, but in addition low-type effort is biased upward. Only the low type earns a rent in this example.

3.3 Comparative statics

Next we study comparative statics in the distortions, in order to relate the model to observed patterns in academic publishing over time.

Scrutiny of obviousness has likely become easier over time. Computerized indexes of articles have made literature searches easier and advances in information technology have made the transmission of papers, especially working papers, more efficient. Today reviewers have the entire literature available and searchable on the internet at all times. Formally, we consider varying the effectiveness of scrutiny (ρ) and the likelihood of the submission being non-obvious (α). As the following proposition shows, increasing either increases effort by the high type; in other words, since neither of these variables impact first-best effort, they reduce distortions.

Proposition 4 (Comparative statics)**.** *The constrained-efficient level of high-type effort e_h is monotone increasing in ρ and α.*

Our model predicts that a lower cost of scrutiny, and the resulting increase in scrutiny, should make papers at the top journals longer, since effort does not need to be biased downward as much. Papers at top journals will be asked for more extensions, robustness checks, and overall length over time as scrutiny improves.[20] Card and DellaVigna (2013) report that papers at the

[20]To see that lower marginal cost of scrutiny increases scrutiny, consider two different scrutiny cost functions $\gamma_1(\rho)$ and $\gamma_2(\rho)$. Let the optimal choice for γ_1 be ρ_1 and the optimal choice for γ_2 be ρ_2. Letting $w(\rho)$ denote the value function of the planner's problem in (4), we then have

$$w(\rho_1) - \gamma_1(\rho_1) \geq w(\rho_2) - \gamma_1(\rho_2)$$
$$w(\rho_2) - \gamma_2(\rho_2) \geq w(\rho_1) - \gamma_2(\rho_1),$$

implying

$$\gamma_1(\rho_2) - \gamma_1(\rho_1) \geq \gamma_2(\rho_2) - \gamma_2(\rho_1)$$

i.e., $\rho_2 > \rho_1$ only if the marginal cost of going from ρ_1 to ρ_2 is higher under γ_1 than under γ_2.

top five economics journals have gotten longer over the last decades, which is consistent with this prediction.

As pointed out in Section 2, higher scrutiny does not necessarily imply higher acceptance rates for non-obvious papers; it only implies that the relative acceptance rates of non-obvious and obvious submissions to top journals must increase. Indeed Card and DellaVigna (2013) document that acceptance rates have fallen at top journals, which is consistent with non-obvious papers facing lower success rates. When the IR constraint binds for the high types, which is often the case, this implies that the rewards to getting an acceptance at a top journal must be rising. This aligns with the experience of the economics discipline which has seen the top five journals rise in importance over time (Heckman and Moktan, 2020).[21]

The comparative static on effort put into obvious ideas (type l), and thus the prediction on the length of articles published in lower-tier journals, is ambiguous. Those papers could get either shorter or longer in response to more scrutiny, depending on parameters. In the example of Figure 2, e_l decreases with ρ so that higher scrutiny makes low type papers shorter. By contrast, Figure 3 depicts an example in which e_l increases with ρ over some range. This example is based on the same parameters as Figure 1, except that $\kappa = 2$ and $l = 0.1$.

The comparative static of high-type effort with respect to changes in α can be interpreted as varying the prior probability that the idea is non-obvious. One proxy for this might be the academic track record of the author. Suppose a journal had a fixed level of scrutiny, independent of the authors' seniority. Then the more senior author (with higher α) would be asked for more development. This is consistent with evidence in Card and DellaVigna (2020) that, conditional on referee recommendations, papers by more prolific

[21] This rise has been alternately called "The Tyranny of the Top Five" by Heckman and Moktan (2020) and "Top5itis" by Serrano (2018).

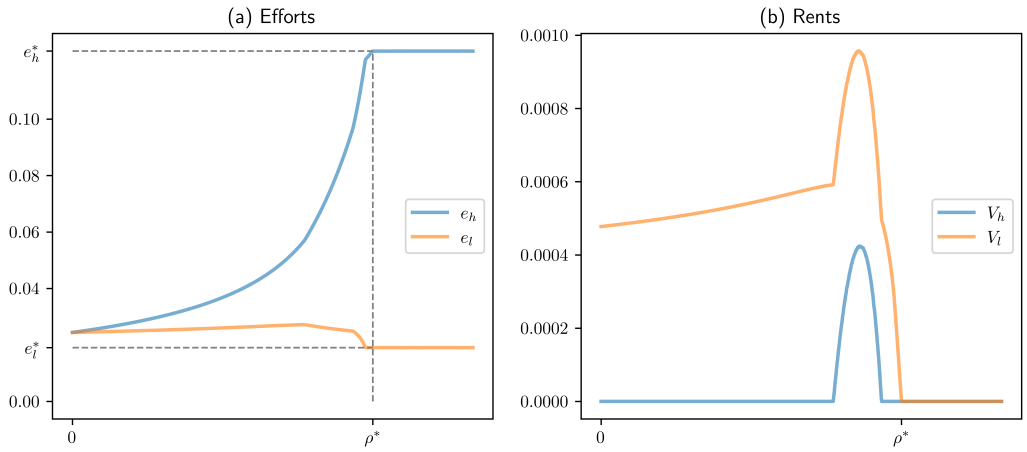

Figure 3: Research efforts and rents in a case where e_l is nonmonotonic

authors receive more citations, suggesting that prolific authors are asked for greater effort, as the model predicts.[22]

4 Discussion

In this section we further discuss two assumptions that have been maintained throughout: that low-type submissions face no scrutiny, and that effort does not impact the signal distribution.

Scrutiny at lower-tier journals. We now briefly discuss under what circumstances setting $x_l = 0$, as we have assumed for expositional purposes, is optimal for the planner and thus without loss of generality. The planner's motive in choosing $x_l > 0$ would be to reduce high-type rents. In cases where a regular solution is always optimal, like Figure 1, it is immediate that such a motive is absent: the high type is on the IR constraint everywhere. Even

[22]Note that they do not find evidence of overciting for prolific authors, consistent with the interpretation that these papers, as a result of receiving higher effort, are indeed of higher quality.

in examples like the one in Figure 2, where the IC constraint binds for the high type for a range of scrutiny, it is still the case that rents for the high type are zero, and therefore $x_l = 0$ must be optimal.[23] The only time that $x_l > 0$ could be optimal would be if the high type could not impersonate the low type and rents are positive, for instance in the particular range of ρ where this occurs in Figure 3.

There are a number of circumstances under which high type rents can be ruled out. The first arises if the cost of effort for low types is sufficiently small relative to the planner's cost of scrutiny. This is because the planner can reduce high-type rents either by scrutinizing low types or by asking them to provide more development effort.[24] Alternatively, $x_l > 0$ can be ruled out if the cost of scrutiny on high types is sufficiently small, so that the optimal high-type scrutiny would be large enough for the high-type IR to be binding (ρ close to ρ^*), or if there is a minimum amount of scrutiny that must be applied by the editors of top-tier journals (for example, to figure out whether to send the paper out for review and which referees to select).

Effort and signal precision. Effort at development might impact the planner's ability to ascertain obviousness, which would give the planner another reason to either encourage or discourage effort by researchers, in intuitive ways. On the one hand, development of ideas might make them clearer and therefore the obviousness better understood. On the other hand, it has been noted, including explicitly in the patent law doctrine, that effort at development might make a non-obvious idea seem obvious.[25] In order to isolate

[23]Examples like Figure 2 are not unusual; indeed for many parameter values where the IC constraint binds for the high type for a range of ρ, there are no high type rents.

[24]The latter is what the planner does in the example of Figure 2 for $\rho < \hat{\rho}$. Notice that, in the example of Figure 3, the parameter κ, which governs the cost of effort, is larger than in the other two examples.

[25]In patent law such a conclusion on obviousness is discouraged by the doctrine of impermissible hindsight. Hindsight bias is more generally a concern in the legal setting;

the incentive effects of effort, we assume that effort has no direct effect on the signals received.[26]

5 Conclusion

We introduce a model of screening for non-obvious ideas, where society benefits from development of non-obvious ideas compared to obvious ones, but non-obvious ideas are more costly to develop. This generates a tension between screening for obviousness and the efficient development of ideas: full development of non-obvious ideas generates rents for obvious ideas. When incentive constraints bind, development of both types of ideas is distorted away from the first best: downward for non-obvious ideas, and upward for obvious ideas. The structure of rewards offered to these heterogeneous ideas is similar to a journal system with tiers of journal quality. The model is also consistent with two-tiered patent systems which have been implemented in some jurisdictions.

When the technology for scrutinizing ideas becomes more efficient, so that scrutiny provides more information about submissions, non-obvious ideas are asked for more development. This is consistent with evidence that top journal articles in economics have become longer. Authors who are seen as more likely to have non-obvious ideas are asked for further development of any given submission they make, consistent with evidence that senior authors are asked to do more.

Certainly many forces are behind these trends. Quantitatively analyzing the importance of the forces outlined in this paper is a potential topic for future research.

see Rachlinski (1998) and also in the patent context specifically Mandel (2006).

[26]Formally, this is embedded in our assumption that p_i depends only on x and θ but not on e.

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APPENDIX

Proof of Proposition 1. We start by establishing the claim that the optimal allocation involves strictly positive effort for the high type and at least first-best effort for the low type.

Lemma 2. *The solution to* (2) *has* $e(l) \geq e^*(l)$ *and* $e(h) > 0$.

Proof. Fix an allocation with $e(l) = \bar{e}(l) < e^*(l)$ and $S(l) = \bar{S}(l)$, and suppose it is optimal. Let $\bar{S} = \sum_i p_i(0,l)\bar{S}_i(l)$ denote the low type's expected reward in this allocation. Consider a marginal increase in the low type's effort accompanied by an increase in the low type's expected reward by $c_e(l, \bar{e}_l)$, which just compensates for the cost increase. By construction, this satisfies the low type's IR and IC constraints while leaving the high type's IR constraint unaffected. It also satisfies the high type's IC constraint: because, by assumption, $p_i(0,l) = p_i(0,h)$, the high type's payoff from reporting the low type is $\sum_i p_i(0,h)\bar{S}_i(l) - c(h, \bar{e}(l)) = \bar{S} - c(h, \bar{e}(l))$, so the change in their payoff from misreporting is $c_e(l, \bar{e}(l)) - c_e(h, \bar{e}(l)) \leq 0$ since $c_{e\theta} > 0$. Let W denote the planner's payoff. The marginal effect of the change in $e(l)$ on the planner's payoff is

$$\left.\frac{dW}{de(l)}\right|_{e(l)=\bar{e}(l)} = (1-\alpha)[l - (1+\delta)c_e(l, \bar{e}(l))] > 0$$

where the inequality follows from the definition of $e^*(l)$ and from $c_{ee} > 0$, implying $l > (1+\delta)c_e(l,e)$ for $e < e(l)^*$. But $dW/de(l) > 0$ contradicts the optimality of the allocation.

Now consider an allocation with $e(h) = 0$, and suppose it is optimal. First we show that this implies $S_i(h) = 0$ for all i. If it were not, then the

contract could be replaced with one that offered $S_i(h) = 0$ for all i and $S(l)$ such that $\sum_i S_i(l) = c(l, e(l))$. Such a contract is IR for both types and IC for l by construction, and must be IC for h since $c(h, e(l)) > c(l, e(l))$ implies that misreporting gives h a strictly negative payoff. Moreover, it saves the planner resources. Given that $e(h) = S_i(h) = 0$, it must be that $x(h) = 0$. For any $x(h) > 0$, replacing $x(h)$ with $x(h) = 0$ saves resources and does not change the researcher's payoffs since $S_i(h)$ is identically zero for every i. Therefore, throughout what follows, we maintain that $S_i(h) = 0$ for all i and $x(h) = 0$.

Fix $e(l) = \bar{e}(l)$ and $S(l) = \bar{S}(l)$. The previous argument implies that $\bar{S}(l)$ must be such that $\sum_i \bar{S}_i(l) = c(l, \bar{e}(l))$, so that the low type's IR and IC constraints both hold with equality. Now consider an alternative allocation with $e(h) = \varepsilon$, $S_i(h) = c(h, \varepsilon)$ for all i, $e(l) = \bar{e}(l)$, and $S_i(l) = \bar{S}_i(l) + c(h, \varepsilon) - c(l, \varepsilon)$ for all i. By construction, this alternative allocation satisfies the high type's IR constraint and the low type's IC constraint; moreover, for $\varepsilon = 0$, it is equivalent to the original allocation. It satisfies the low type's IR constraint since $c_\theta > 0$. The high type's IC constraint is satisfied if

$$\begin{aligned}\sum_i p_i(0,h)S_i(h) - c(h,e(h)) = 0 \geq \sum_i p_i(0,h)S_i(l) - c(h,e(l)) \\ = \sum_i p_i(0,l)\bar{S}_i(l) - c(h,\bar{e}(l)) + c(h,\varepsilon) - c(l,\varepsilon) \\ = c(h,\varepsilon) - c(l,\varepsilon) - (c(h,\bar{e}(l)) - c(l,\bar{e}(l))),\end{aligned}$$

which, because $c_\theta > 0$, holds for $\varepsilon < \bar{e}(l)$, and thus for ε sufficiently small since $\bar{e}(l) \geq e^*(l) > 0$ by the first part of the lemma. The marginal effect on the planner's payoff from raising ε slightly above zero is

$$\left.\frac{dW}{d\varepsilon}\right|_{\varepsilon=0} = \alpha\left[h - (1+\delta)c_e(h,0)\right] - (1-\alpha)\delta[c_e(h,0) - c_e(l,0)] > 0$$

where the inequality follows from $c_e(\theta, 0) = 0$. But $dW/d\varepsilon > 0$ contradicts the optimality of the original allocation which is equivalent to $\varepsilon = 0$. □

By Lemma 2, $e(h) > 0$ at the solution to the planner's problem. IR then implies that the high type's expected reward is strictly positive: $\sum_i p_i(x(h), h)S_i(h) \geq c(h, e(h)) > 0$. The rest of the proof proceeds as follows. We first derive a result about the optimal structure of the high type's reward. Then we show that the conditions for that result to apply must hold at the optimum.

Lemma 3. *Fix scrutiny $x(h) = x > 0$. To minimize the low type's reward from misreporting, $\sum_i p_i(x, l)S_i(h)$, conditional on giving an expected reward of $\bar{S} > 0$ to the high type, the planner optimally sets $S_k(h) = \bar{S}/p_k(x, h) > 0$ and $S_i(h) = 0$ for $i \neq k$.*

Proof. The problem is

$$\min_{S(h)} \sum_i p_i(x, l)S_i(h)$$

subject to

$$\sum_i p_i(x, h)S_i(h) = \bar{S} \tag{11}$$

$$S_i(h) \geq 0, \quad i = 1, \ldots, n \tag{12}$$

Because we fix x we can drop the dependence on x and let $p_i^\theta = p_i(x, \theta)$. The Lagrangian of the problem is

$$\mathcal{L} = \sum_{i=1}^{n} p_i^l S_i(h) + \lambda[\bar{S} - \sum_{i=1}^{n} p_i^h S_i(h)] - \sum_{i=1}^{n} \mu_i S_i(h),$$

where λ is the Lagrange multiplier associated with constraint (11) and μ_i is the multiplier associated with the non-negativity constraint on $S_i(h)$.

Claim 1. $S_i(h) > 0$ for at most one i. Differentiating $\mathcal{L}$ with respect to $S_i(h)$ yields the first-order condition $p_i^l - \lambda p_i^h = \mu_i$. If $S_i(h) > 0$, then $\mu_i = 0$, and thus

$$\frac{p_i^l}{p_i^h} = \lambda.$$

But since, by assumption, no two signals have the same likelihood ratio, this can hold for at most one i.

Claim 2. $S_i(h) > 0$ *for at least one* i. Suppose $S_i(h) = 0$ for all i. Then, $\sum_i p_i^h S_i(h) = 0 < \bar{S}$, violating (11).

Together these two claims imply that there is exactly one i for which $\mu_i = 0$. Let k be the signal with the lowest likelihood ratio, i.e., $k = \arg\min_i p_i^l/p_i^h$.

Claim 3. $S_k > 0$. Suppose to the contrary that $S_k = 0$. Then $\mu_k > 0$ and $\mu_i = 0$ for some $i \neq k$. Hence,

$$\frac{p_k^l}{p_k^h} = \lambda + \frac{\mu_k}{p_k^h} > \lambda = \frac{p_i^l}{p_i^h},$$

a contradiction with k having the lowest likelihood ratio. □

Lemma 3 implies that, fixing effort levels at $e(h) > 0$ and $e(l) \geq e^*(l)$, and fixing the expected reward to the high type at $\bar{S} > 0$, if scrutiny is strictly positive and the low type's IC constraint is binding, it is optimal for the planner to load all of the high type's reward on signal k. The next two lemmas show that the low type's IC must indeed be binding and that high-type scrutiny must be strictly positive at the optimum.

Lemma 4. *Fix* $e(h) > 0$, $e(l) \geq e^*(l)$, *and* $\sum_i p_i(x(h), h)S_i(h) = \bar{S} > 0$. *The optimal combination of* $S(l)$, $S(h)$, *and* $x(h)$ *is such that the low type's IC constraint binds.*

Proof. Suppose to the contrary that the low type's IC is slack at the optimal allocation, i.e.,

$$\sum_i p_i(0, l)S_i(l) - c(l, e(l)) > \sum_i p_i(x(h), l)S_i(h) - c(l, e(h)). \tag{13}$$

If the low type's IR is also slack, $\sum_i p_i(0, l)S_i(l) - c(l, e(l)) > 0$, then lowering $S_i(l)$ for some i where $S_i(l) > 0$ increases the planner's payoff, immediately leading to a contradiction. (Together with $e(l) > 0$, IR implies that there

must be such an i.) Thus, suppose in what follows that the low type's IR constraint is binding.

There are two cases. If $x(h) = 0$, then $\sum_i p_i(0,l)S_i(l) - c(l,e(l)) = 0 > \sum_i p_i(0,l)S_i(h) - c(l,e(h)) > \sum_i p_i(0,h)S_i(h) - c(h,e(h))$, where the first inequality is due to (13) and the second to the assumptions that $p_i(0,l) = p_i(0,h)$ and $c_\theta > 0$. But this implies that the allocation violates the high type's IR, again yielding a contradiction.

If $x(h) = x > 0$, let $p_i \equiv p_i(x,h)$, and consider a small decrease in scrutiny to $x(h) = x' < x$. Let $p_i' = p_i(x',h)$, and change the high type's reward to $S_i'(h) = p_i S_i(h)/p_i'$ for all i for which $p_i' > 0$. For $x' - x$ sufficiently small, whenever $p_i > 0$, by continuity $p_i' > 0$, so $\sum_i p_i' S_i'(h) = \sum_i p_i S_i(h)$. Thus, the high type's expected reward remains unchanged (and the high type's IR and IC continue to be satisfied). By continuity, for $x' - x$ sufficiently small, $p_i(x',\theta)$ is close to $p_i(x,\theta)$. Hence, the low type's IC remains satisfied. But because $x' < x$ the planner saves on scrutiny costs, contradicting the optimality of the allocation. □

Lemma 5. *The optimal contract has $x(h) > 0$.*

Proof. Consider an allocation with $x(h) = 0$ and suppose it is optimal. Let $\bar{S} \equiv \sum_i p_i(0,h)S_i(h) > 0$ denote the high type's expected reward in this allocation. Now consider an alternative allocation with $x(h) \geq 0$, $S_i(h) = 0$ for all $i \neq k$ and $S_k(h) = \bar{S}/p_k(x(h),h)$. By construction, for $x(h) = 0$ this alternative allocation yields the same payoff to the planner as the original allocation: the high type's expected reward is unchanged irrespective of $x(h)$, and the low type's expected reward from misreporting is unchanged for $x(h) = 0$ since $p_i(0,l) = p_i(0,h)$, so the low type's IC continues to hold.

Lemma 4 implies that the low type's IC is binding at the optimum, and hence also with the alternative allocation for $x(h) = 0$. Thus, the low type's expected payoff is equal to the payoff from misreporting. Under the al-

ternative allocation, the low type's payoff from misreporting is, for general $x(h) \geq 0$,

$$\sum_i p_i(x(h), l)S_i(h) - c(l, e(h)) = \frac{p_k(x(h), l)}{p_k(x(h), h)}\bar{S} + c(l, e(l)) - c(l, e(h)).$$

The marginal effect on the planner's payoff of increasing $x(h)$ slightly above zero and adjusting $S(l)$ such that the low type's IC remains binding is therefore

$$\left.\frac{dW}{dx(h)}\right|_{x(h)=0} = -\alpha - (1-\alpha)\delta\frac{\partial}{\partial x}\left(\frac{p_k(0,l)}{p_k(0,h)}\right)\bar{S} > 0,$$

where the inequality follows from Assumption 2. But this contradicts the optimality of the allocation with $x(h) = 0$. □

This establishes the claim that $x(h) > 0$. In addition, Lemmas 2, 3, 4, and 5 together imply that the optimal contract has all rewards loaded on signal k. □

Proof of Lemma 1. Consider the allocation $(e_l, e_h, V_l, V_h) = (e_l^*, e_h^*, 0, 0)$. Since the allocation is first best, it solves the planner's problem if it satisfies the IC constraints. The high type's IC holds if

$$0 \geq -(c(h, e_l^*) - c(l, e_l^*)),$$

which is always satisfied because $c_\theta > 0$. The low type's IC holds if

$$0 \geq (1-\rho)c(h, e_h^*) - c(l, e_h^*),$$

or $\rho \geq \rho^*$. □

Proof of Proposition 2. We first show that the low type's IC must bind.

Lemma 6. *The low type's IC is binding at the planner's optimum for $\rho < \rho^*$.*

Proof. Suppose to the contrary that the low type's IC is slack. Then the low type's IR must be binding (otherwise reduce V_l). Thus,

$$V_l = 0 > (1-\rho)(V_h + c(h, e_h)) - c(l, e_h). \tag{14}$$

The high type's IC becomes $V_h \geq -(c(h, e_l)) - c(l, e_l))$, which is implied by the high type's IR. Thus IR must be binding for both types, $V_l = V_h = 0$, in which case the planner's payoff is maximized at the first-best efforts, $e_l = e_l^*$ and $e_h = e_h^*$. But because $(1-\rho)c(h, e_h^*) - c(l, e_h^*) > 0$ for $\rho < \rho^*$, this contradicts (14). □

Now fix an allocation $(\bar{e}_h, \bar{e}_l, \bar{V}_h, \bar{V}_l)$ with $\bar{e}_h \geq e_h^*$ and suppose it is optimal. By Lemma 6, the low type's IC constraint is binding, so $\bar{V}_l = (1-\rho)[\bar{V}_h + c(h, \bar{e}_h)] - c(l, \bar{e}_h) > 0$, where the inequality follows from $\rho < \rho^*$ and multiplicative separability (Assumption 1), implying

$$1 - \rho > 1 - \rho^* = \frac{c(l, e_h^*)}{c(h, e_h^*)} = \frac{\kappa(l)\psi(e_h^*)}{\kappa(h)\psi(e_h^*)} = \frac{\kappa(l)\psi(e_h)}{\kappa(h)\psi(e_h)} = \frac{c(l, e_h)}{c(h, e_h)}$$

for all $e_h > 0$. Consider a marginal change in $\bar{e}_h$ while holding e_l and V_h constant. Notice that the high-type's IR and IC constraints are unaffected since e_l and V_h are unchanged; moreover, the low type's IR continues to hold since it is slack for $e_h = \bar{e}_h$. The effect on the planner's payoff W is

$$\left.\frac{dW}{de_h}\right|_{e_h = \bar{e}_h} = \alpha(h - (1+\delta)c_e(h, \bar{e}_h)) - (1-\alpha)\delta \left.\frac{dV_l}{de_h}\right|_{e_h = \bar{e}_h}.$$

The first term is non-positive since $h \leq (1+\delta)c_e(h, e_h)$ for $e_h \geq e_h^*$. We have

$$\left.\frac{dV_l}{de_h}\right|_{e_h = \bar{e}_h} = (1-\rho)c_e(h, \bar{e}_h) - c_e(l, \bar{e}_h).$$

Under multiplicative separability,

$$1 - \rho^* = \frac{c(l, e_h^*)}{c(h, e_h^*)} = \frac{\kappa(l)\psi(e_h^*)}{\kappa(h)\psi(e_h^*)} = \frac{\kappa(l)\psi'(\bar{e}_h)}{\kappa(h)\psi'(\bar{e}_h)} = \frac{c_e(l, \bar{e}_h)}{c_e(h, \bar{e}_h)},$$

implying $dV_l/de_h = 0$ for $\rho = \rho^*$ and thus $dV_l/de_h > 0$ for $\rho < \rho^*$. Therefore $dW/de_h < 0$ for $\rho < \rho^*$ which contradicts optimality of $\bar{e}_h \geq e_h^*$. □

Proof of Proposition 3. Fix an allocation $(\bar{e}_h, \bar{e}_l, \bar{V}_h, \bar{V}_l)$ with $\bar{e}_l = e_l^*$ and suppose it is optimal. (By Lemma 2, which does not rely on scrutiny being optimally chosen, $e_l < e_l^*$ can never be optimal.) Since $\rho < \rho^*$, Lemma 6 implies that the low type's IC is binding, so $V_l = (1-\rho)(\bar{V}_h + c(h, \bar{e}_h)) - c(l, \bar{e}_h) > 0$ at the solution to the planner's problem.

Define $\hat{e}_h(e_l)$ as the value of e_h solving, for a given e_l,

$$(1-\rho)c(h, e_h) - c(l, e_h) = c(h, e_l) - c(l, e_l). \tag{15}$$

Note that because, by assumption, condition (10) is violated, $(1-\rho)c(h, \tilde{e}(\rho)) - c(l, \tilde{e}(\rho)) > c(h, e_l^*) - c(l, e_l^*)$. Under multiplicative separability (Assumption 1), the left-hand side of (15) is increasing in e_h for $\rho < \rho^*$, and therefore $\hat{e}_h(e_l^*) < \tilde{e}(\rho)$. There are two cases:

(a) $\bar{e}_h > \hat{e}_h(e_l^*)$. Then $\bar{V}_h > 0$ because the high type's IC would be violated for $\bar{V}_h = 0$:

$$\bar{V}_l - (c(h, e_l^*) - c(l, e_l^*)) = (1-\rho)c(h, \bar{e}_h) - c(l, \bar{e}_h) - (c(h, e_l^*) - c(l, e_l^*)) > 0,$$

where the inequality follows from $\bar{e}_h > \hat{e}_h(e_l^*)$ and the definition of $\hat{e}_h(e_l^*)$. This implies that IC must be binding and IR slack for the high type. Thus V_l and V_h solve a system of two equations in two unknowns given by (7) and (8) holding with equality, yielding

$$V_l = \frac{(1-\rho)c(h, e_h) - c(l, e_h) - (1-\rho)[c(h, e_l) - c(l, e_l)]}{\rho} \tag{16}$$

$$V_h = \frac{(1-\rho)c(h, e_h) - c(l, e_h) - [c(h, e_l) - c(l, e_l)]}{\rho}. \tag{17}$$

The marginal effect of increasing e_l on the planner's payoff is

$$\left.\frac{dW}{de_l}\right|_{e_l = e_l^*} = -\alpha \left.\frac{\partial V_h}{\partial e_l}\right|_{e_l = e_l^*} + (1-\alpha)\left[l - (1+\delta)c_e(l, e_l^*) - \delta \left.\frac{\partial V_l}{\partial e_l}\right|_{e_l = e_l^*}\right].$$

We have

$$\frac{\partial V_l}{\partial e_l} = (1-\rho)\frac{\partial V_h}{\partial e_l} = -\frac{(1-\rho)[c_e(h,e_l) - c_e(l,e_l)]}{\rho} < 0$$

for $e_l > 0$ because $c_{e\theta} > 0$. Moreover, by definition of e_l^*, $l - (1+\delta)c_e(l,e_l^*) = 0$. Therefore, $dW/de_l > 0$, which contradicts optimality of $\bar{e}_l = e_l^*$.

(b) $\bar{e}_h \leq \hat{e}_h(e_l^*)$. Then

$$(1-\rho)c(h,\bar{e}_h) - c(l,\bar{e}_h) - (c(h,e_l^*) - c(l,e_l^*)) \leq 0$$

by the definition of $\hat{e}_h(e_l^*)$, so the high type's IC is satisfied for $V_h = 0$, implying that the high type's IR must be binding (otherwise reduce V_h), and $\bar{V}_h = 0$.

Suppose first $\bar{e}_h < \hat{e}_h(e_l^*)$. Then the high type's IC does not bind and continues to hold if e_h is raised marginally. The IR constraints are unaffected, and V_l adjusts by $\partial V_l/\partial e_h = (1-\rho)c_e(h,\bar{e}_h) - c_e(l,\bar{e}_h)$. At $e_h = \tilde{e}(\rho)$,

$$\left.\frac{dW}{de_h}\right|_{e_h=\tilde{e}(\rho)} = \alpha[h - (1+\delta)c_e(h,\tilde{e}(\rho))] \\ - (1-\alpha)\delta[(1-\rho)c_e(h,\tilde{e}(\rho)) - c_e(l,\tilde{e}(\rho))] = 0.$$

Moreover, $c_{ee} > 0$ and multiplicative separability imply that dW/de_h is decreasing in e_h. Thus, at $\bar{e}_h < \hat{e}_h(e_l^*) < \tilde{e}(\rho)$, we have $dW/de_h > 0$, contradicting optimality.

Suppose instead $\bar{e}_h = \hat{e}_h(e_l^*)$, so that the high type's IC binds. Consider a marginal increase in e_l accompanied by a marginal change in e_h by $\hat{e}_h'(e_l^*)$, which keeps the high type's IC binding, while holding V_h fixed

(at zero). The marginal effect on the planner's payoff is

$$\left.\frac{dW}{de_l}\right|_{e_l=e_l^*} = \alpha\hat{e}_h'(e_l^*)\left[h-(1+\delta)c_e(h,\hat{e}_h(e_l^*))\right] + (1-\alpha)\left[l-(1+\delta)c_e(l,e_l^*)-\delta\hat{e}_h'(e_l^*)\left.\frac{\partial V_l}{\partial e_h}\right|_{e_h=\hat{e}_h(e_l^*)}\right].$$

We have $l-(1+\delta)c_e(l,e_l^*)=0$ by definition of e_l^*, and

$$\left.\frac{\partial V_l}{\partial e_h}\right|_{e_h=\hat{e}_h(e_l^*)} = (1-\rho)c_e(h,\hat{e}_h(e_l^*)) - c_e(l,\hat{e}_h(e_l^*)).$$

Moreover, by the implicit function theorem,

$$\hat{e}_h'(e_l) = \frac{c_e(h,e_l)-c_e(l,e_l)}{(1-\rho)c_e(h,e_h)-c_e(l,e_h)} > 0$$

for $e_l > 0$, $e_h > 0$, and $\rho < \rho^*$. Because $\bar{e}_h = \hat{e}_h(e_l^*) < \tilde{e}(\rho)$, the same argument as above then implies $dW/de_l > 0$, which contradicts optimality of $\bar{e}_l = e_l^*$.

This establishes the claim that $\bar{e}_l \leq e_l^*$ cannot be part of an optimal allocation. □

Proof of Proposition 4. The constrained optimal efforts e_l and e_h depend on the set of constraints that are binding at a given $\rho < \rho^*$. For the low type, the IC constraint is always binding by Lemma 6 and the IR constraint slack as shown in the proof of Proposition 2. For the high type, there are three possible cases:

(a) Only the IR constraint binds for the high type.

(b) Only the IC constraint binds for the high type.

(c) The IR and IC constraint both bind for the high type.

Define X_a to be the set of ρ where only the high-type IR and low-type IC constraint bind. Define X_b to be the set of ρ where only the high-type IC and low-type IC constraint bind. Define $X_c = (X_a \cup X_b)^{\complement}$ to be the set of ρ where only the high-type IC, high-type IR, and low-type IC constraint bind.

Since, if constraints are slack, they must remain slack for small changes in ρ, the first two can be described as a union of (disjoint) open intervals. X_c is therefore closed. We first characterize the value of e_h that solves the planner's problem in each of these regions via first-order conditions:

(a) If $\rho \in X_a$, then $e_l = e_l^*$, $V_h = 0$, $V_l = (1-\rho)c(h, e_h) - c(l, e_h)$, and e_h solves the first-order condition $f_a = 0$, where

$$f_a = \alpha[h - (1+\delta)c_e(h, e_h)] - (1-\alpha)\delta[(1-\rho)c_e(h, e_h) - c_e(l, e_h)]. \quad (18)$$

(b) If $\rho \in X_b$, then the rents V_l and V_h are obtained by solving the system of two equations in two unknowns given by the low-type and high-type IC constraints, yielding (16) and (17), reproduced here for convenience,

$$V_l = \frac{(1-\rho)c(h, e_h) - c(l, e_h) - (1-\rho)[c(h, e_l) - c(l, e_l)]}{\rho}$$
$$V_h = \frac{(1-\rho)c(h, e_h) - c(l, e_h) - [c(h, e_l) - c(l, e_l)]}{\rho},$$

and e_h solves the first-order condition $f_b = 0$, where

$$f_b = \alpha[h - (1+\delta)c_e(h, e_h)] - \frac{\delta\left((1-\rho)c_e(h, e_h) - c_e(l, e_h)\right)}{\rho}. \quad (19)$$

(c) If $\rho \in X_c$, then $V_h = 0$, $V_l = (1-\rho)c(h, e_h) - c(l, e_h)$, and e_l and e_h solve

$$\max_{e_l, e_h} \alpha[he_h - (1+\delta)c(h, e_h)] + (1-\alpha)[le_l - (1+\delta)c(l, e_l) - \delta((1-\rho)c(h, e_h) - c(l, e_h))] \quad (20)$$

$$\text{subject to} \quad (1-\rho)c(h, e_h) - c(l, e_h) = c(h, e_l) - c(l, e_l). \quad (21)$$

Combining the first-order conditions to eliminate the Lagrange multiplier on the constraint and defining $\hat{e}_l(e_h)$ as the value of e_l solving (21) for a given e_h, the optimal e_h solves $f_c = 0$, where

$$f_c = \alpha[h - (1+\delta)c_e(h, e_h)] - (1-\alpha)\left[(1-\rho)c_e(h, e_h) - c_e(l, e_h)\right]\Psi \quad (22)$$

with

$$\Psi = \delta - \frac{l - (1+\delta)c_e(l, \hat{e}_l(e_h))}{c_e(h, \hat{e}_l(e_h)) - c_e(l, \hat{e}_l(e_h))}.$$

We now show that, in each region, there is a unique solution. For regions (a) and (b), this follows from the fact that f_a and f_b are strictly decreasing in e_h because $c_{ee} > 0$ and $(1-\rho)c_{ee}(h, e_h) - c_{ee}(l, e_h) > 0$ for $\rho < \rho^*$ by Assumption 1. For region (c), note that there exists $\underline{e}$ such that $\Psi > 0$ for all $e_h > \underline{e}$ because $l < (1+\delta)c_e(l, e_l)$ for $e_l > e_l^*$ and $\hat{e}_l$ is monotone increasing in e_h: by the implicit function theorem,

$$\frac{d\hat{e}_l}{de_h} = \frac{(1-\rho)c_e(h, e_h) - c_e(l, e_h)}{c_e(h, e_l) - c_e(l, e_l)} > 0$$

for $e_l > 0$, $e_h > 0$, and $\rho < \rho^*$. The set of $e_h > \underline{e}$ contains all solutions because, by Proposition 2, the optimal e_h is distorted down, which requires $\Psi > 0$. At every $e_h > \underline{e}$ solving $f_c = 0$, f_c is downward-sloping: we have

$$\frac{\partial f_c}{\partial e_h} = -\alpha c_{ee}(h, e_h) - (1-\alpha)\Bigg[\Big((1-\rho)c_{ee}(h, e_h) - c_{ee}(l, e_h)\Big)\Psi + \Big((1-\rho)c_e(h, e_h) - c_e(l, e_h)\Big)\frac{d\Psi}{de_h}\Bigg],$$

$\Psi > 0$ for $e_h > \underline{e}$, and

$$\begin{aligned}\frac{d\Psi}{de_h} &= -\frac{d\hat{e}_l}{de_h}\left[\frac{-(1+\delta)c_{ee}(l, \hat{e}_l(e_h))[c_e(h, \hat{e}_l(e_h)) - c_e(l, \hat{e}_l(e_h))]}{[c_e(h, \hat{e}_l(e_h)) - c_e(l, \hat{e}_l(e_h))]^2}\right. \\ &\qquad \left. - \frac{\Big(l - (1+\delta)c_e(l, \hat{e}_l(e_h))\Big)[c_{ee}(h, \hat{e}_l(e_h)) - c_{ee}(l, \hat{e}_l(e_h))]}{[c_e(h, \hat{e}_l(e_h)) - c_e(l, \hat{e}_l(e_h))]^2}\right] \\ &= \frac{d\hat{e}_l}{de_h}\left[\frac{l[c_{ee}(h, \hat{e}_l(e_h)) - c_{ee}(l, \hat{e}_l(e_h))]}{[c_e(h, \hat{e}_l(e_h)) - c_e(l, \hat{e}_l(e_h))]^2}\right]\end{aligned}$$

because Assumption 1 implies $c_e(\theta, e)c_{ee}(\theta', e) = c_{ee}(\theta, e)c_e(\theta', e)$ for all θ, θ'. It then follows from $c_\theta > 0$ and Assumption 1, implying $c_{ee\theta} > 0$, that $d\Psi/de_h > 0$, and hence that $\partial f_c/\partial e_h < 0$ for $e_h > \underline{e}$. Finally, because f_c is a continuous function, there can only be one $e_h > \underline{e}$ such that $f_c = 0$, given that $\partial f_c/\partial e_h < 0$ for all $e_h > \underline{e}$.

Let X_c^I denote the interior of X_c. We now show that, for each of the open sets X_a, X_b, and X_c^I, the value of e_h solving the planner's problem is increasing in ρ and α. By the implicit function theorem, in region j,

$$\frac{de_h}{dz} = -\frac{\partial f_j/\partial z}{\partial f_j/\partial e_h}, \qquad j = a, b, c,$$

where $z \in \{\rho, \alpha\}$ is the parameter of interest. Because the second-order condition implies that $\partial f_j/\partial e_h < 0$ at the optimal e_h, the sign of de_h/dz equals the sign of $\partial f_j/\partial z$. We have:

(a) for $\rho \in X_a$,

$$\frac{\partial f_a}{\partial \rho} = (1-\alpha)\delta c_e(h, e_h) > 0$$
$$\frac{\partial f_a}{\partial \alpha} = h - (1+\delta)c_e(h, e_h) + \delta[(1-\rho)c_e(h, e_h) - c_e(l, e_h)] > 0,$$

where the inequalities follow from $c_e > 0$ and the fact that, by Assumption 1, we have $(1-\rho)c_e(h, e_h) - c_e(l, e_h) > 0$ for $\rho < \rho^*$, as well as the fact that, by Proposition 2 we have $e_h < e_h^*$ and thus $h > (1+\delta)c_e(h, e_h)$;

(b) for $\rho \in X_b$,

$$\frac{\partial f_b}{\partial \rho} = \frac{\delta[c_e(h, e_h) - c_e(l, e_h)]}{\rho^2} > 0$$
$$\frac{\partial f_b}{\partial \alpha} = h - (1+\delta)c_e(h, e_h) > 0,$$

where the inequalities follow from $c_{e\theta} > 0$ and again from the fact that Proposition 2 implies $h > (1+\delta)c_e(h, e_h)$;

(c) for $\rho \in X_c^I$,

$$\frac{\partial f_c}{\partial \rho} = -(1-\alpha)\left[-c_e(h,e_h)\Psi + [(1-\rho)c_e(h,e_h) - c_e(l,e_h)]\frac{d\Psi}{d\rho}\right] > 0$$
$$\frac{\partial f_c}{\partial \alpha} = h - (1+\delta)c_e(h,e_h) + [(1-\rho)c_e(h,e_h) - c_e(l,e_h)]\Psi > 0.$$

To establish these inequalities, note that

$$\frac{d\Psi}{d\rho} = \frac{d\hat{e}_l}{d\rho}\left[\frac{l[c_{ee}(h,\hat{e}_l(e_h)) - c_{ee}(l,\hat{e}_l(e_h))]}{[c_e(h,\hat{e}_l(e_h)) - c_e(l,\hat{e}_l(e_h))]^2}\right]$$

and, again applying the implicit function theorem,

$$\frac{d\hat{e}_l}{d\rho} = -\frac{c(h,e_h)}{c_e(h,e_l) - c_e(l,e_l)} < 0.$$

Because $c_{e\theta} > 0$ and $c_{ee\theta} > 0$, we thus have $d\Psi/d\rho < 0$. The fact that $\Psi > 0$ at the optimal e_h completes the argument.

Therefore e_h is increasing on $X_a \cup X_b \cup X_c^I$. To see that e_h is increasing throughout, let $y = (e_l, e_h, V_l, V_h)$ so that the planner's problem can be written as $\max_y W$ subject to $y \in \Gamma(\rho)$. Note that the constraint set $\Gamma(\rho)$ forms a continuous correspondence. It is upper hemicontinuous since it has a closed graph. It is lower hemicontinuous since the constraints are intervals, so all points $y \in \Gamma(\rho)$ are either on the interior, and hence there trivially is a sequence $y_n \to y$ with $y_n \in \Gamma(\rho_n)$ and $y_n \to y$, or y is on the (continuous) boundary of the constraint set, and such a sequence can be described by $y_n \in \text{Boundary}(\Gamma(\rho_n))$. Therefore the solution correspondence is continuous in ρ by the Theorem of the Maximum, and in turn is a continuous function since it is single valued, as established previously.

The continuous solution e_h is therefore increasing almost everywhere since the closure points of X_c are locally isolated points. But a continuous function that is increasing almost everywhere must be an increasing function. □

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